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# A model for determining the optimal base stock level when the lead time has a change of distribution property

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**Abstract-**A new type of inventory model using the so called "Change of Distribution property" is analyzed in this paper. Base Stock system for patient customers is a special type of ordering mechanism in inventory control theory. The inventory process begins with an initial inventory of B units. Whenever a customer's order for 'r' units is received, an inventory replenishment order of 'r' units is placed. Replenishment orders are fulfilled after the lead time L. If the demand exceeds the stock on hand, then assume that the customer will not cancel the order but await the arrival of sufficient stock. Here we assume that L is a random variable and it satisfies the Change of Distribution property and so the distribution undergoes a parametric change after the truncation point. Assuming that the truncation point is a random variable which has the mixed exponential distribution, the optimal value of Base Stock is derived.

**Index Terms-** Base Stock, Lead time, Change of Distribution propertyand Truncation point.

## 1. INTRODUCTION

In inventory control theory, one of the important and fame model is Base Stock system for patient customers and these models have been discussed by many authors. Base Stock system for patient customers is an interesting method of ordering mechanism. The inventory initially begins with 'B' number of units. Whenever a customer order for 'r' units is received, an inventory replenishment order for 'r' units is placed immediately. Replenishment orders are filled after the lead time 'L'. The customer's demand is met with as far as possible from the supply on hand. If the total unfilled customer's demand exceeds the supply on hand, then assume that the customer will not cancel the orders but the customers will wait till their requirement is fulfilled. The sum of inventory on hand and order placed is constant in time and equal to 'B', called as Base Stock.

The detailed study of the Base Stock system for patient customers has been discussed by Gaver (1959). The very basic model for base stock system has been discussed by Hanssman (1962).

In the sense that after the truncation point the lead time takes parametric change. The change point known as the truncation point and is also assumed to be a random variable. The very basic concept of Change of Distribution property was discussed by Stangl (1995). Ramanarayanan et.al (1998) have discussed the model in which the lead time was assumed to be a random variable. Suresh Kumar (2006) has used this concept in the stock model approach.

Sachithanantham et. al (2008) have discussed the modified version of the Base Stock model in which the lead time is assumed to be a random variable and which satisfies the so called Change of distribution property.

Henry et. al (2011) have discussed the base stock system for patient customers with the assumptions that the lead time random variable is continuous and it undergoes the Change of distribution property with the assumption that the truncation point itself a random variable, which follows exponential with parameter $\lambda$.

The Change of distribution property refers that a random variable takes different probability function after the certain point known as truncation point.

In this paper, it is assumed that the lead time random variable has Change of distribution property and the change point itself a random variable, which has the mixed exponential distribution. Under this assumption, the optimal Base Stock level is derived.

## 2. NOTATIONS

| | | |
|---|---|---|
| B | : | The Base Stock level. |
| L | : | A random variable denoting the lead time with the pdf is k(.). |
| U | : | Random variable denoting the inter-arrival times between successive demands during the lead time with Pdf g(.) and cdf G(.). |
| $\lambda$ | : | Parameter of inter arrival time distribution. |

$G_n(.)$ : The $n^{th}$ convolution of G(.).
$X_i$ : A random variable denoting the magnitude of demand at the $i^{th}$ demand epoch with pdf f(.) and F(.) is the cdf and $X_i \sim \exp(\mu)$, i = 1,2,3,…..n.
μ : Parameter of demand distribution.
$F_k(.)$ : The $k^{th}$ convolution of F(.).
h : Inventory holding cost / unit / time.
d : Shortage cost / unit / time.

## 3. PROBLEM FORMATION

Consider the Base Stock system, in which let X be the amount of demand during the lead time L. If f(x) is the probability density function of demand, then the Expected Cost per unit time of overages and shortages attributed to the inventory on ground is,

$$E(C) = h\int_0^B (B-X)\,f(x)dx + d\int_B^\infty (X-B)\,f(x)dx$$

Therefore the optimal Base Stock $\hat{B}$ , can be obtained by using $\frac{dE(C)}{dB} = 0$. It follows that $F(\hat{B}) = \frac{d}{h+d}$ (1)

Here F(x) is distribution of f(x).
If there are N demand epochs during L and N is a random variable then the probability that there are exactly 'n' demand during L is

$$P[N = n/L] = G_n(L) - G_{n+1}(L) \text{ [from the renewal theory arguments]}$$

Let X be the total demand during L, say X = $X_1+X_2+$ …..$+X_N$ , then the total demand is at the most x, during L, is given by

$$P[X \le x] = \sum_{n=0}^{\infty} P[X_1 + X_2 + \cdots + X_n \le x/N = n]\,.\,P(N = n/L)$$

$$= \sum_{n=0}^{\infty}[G_n(L) - G_{n+1}(L)]\,.\,F_n(x)$$

Therefore, the expected cost is,

$$E(C) = h\int_0^B (B-X)\,dP(X \le x) + d\int_B^\infty (X-B)\,dP(X \le x)$$

Thus the optimal Base Stock is given by

$$F(\hat{B}) = \frac{d}{h+d} = P[X \le B]$$

Hence, $\frac{d}{h+d} = \sum_{n=0}^{\infty}[G_n(L) - G_{n+1}(L)]\,.\,F_n(B)$ (2)

In the above, we assume that the lead time L be a continuous random variable with probability density function K(.).
For the sake of convenience, take L= Y.
Therefore the optimal Base Stock is given by

$$\sum_{n=1}^{\infty} F_n(B)\int_0^\infty [G_n(L) - G_{n+1}(L)]k(y)dy = \frac{d}{h+d} \quad (3)$$

## 4. DEFINITIONS AND ASSUMPTIONS

### *4.1. Definition*

4.1.1.*Setting the Clock Back to Zero property (SCBZ property)*

The special property known as Setting the Clock Back to Zero property (SCBZ property)is due toRajaRao. B and Talwaker[4].The family of life distributions $\{f(x,\theta),\ x \geq 0,\ \theta \in \Omega\}$ is said to have be ' SCBZ property' is the form of $f(x,\theta)$ remains unchanged except for value of the parameter.

i.e., $f(x,\theta) \rightarrow f(x,\theta^*)$ where $\theta^* \in \Omega$

Under the following three operations

(i) Truncating original distribution of some point $X_0 \geq 0$

(ii) Considering the observable distribution for life time $X \geq X_0$ and

(iii) Changing the origin by means of the transformation given by $X_1 = -X_0$ , $X_1 \geq 0$, where $X_0$ is a Truncation point.

### *4.2. Assumptions*

1. The lead time is continuous random variable and its probability density function undergoes change of distribution property after the truncation point. Here the pdf of lead time random variable is

$$\boldsymbol{f(x)} = \begin{cases} \boldsymbol{f_1(x)} & \boldsymbol{, if\ X \leq X_0} \\ \boldsymbol{f_2(x)} & \boldsymbol{, if\ X > X_0} \end{cases}$$

   Where $X_0$ is a Truncation point.

2. The truncation point is a random variable, which follows Uniform distribution
   Hence, $f(x) = \boldsymbol{f_1}(x)P(X \leq x_0) + \boldsymbol{f_2}(x)P(X > x_0)$
   Here $x_0 \sim U(\alpha,\beta)$
3. The inter-arrival times between successive demand epochs are i.i.d random variables and are assumed to be followed as exponential with parameter $\lambda$.

## 5. MAIN RESULTS

In this paper, an improvised model for base stock system for patient customers is discussed with the assumption that the lead time random variable has Change of distribution. It is also assumed that the truncation point itself a random variable, which has Uniform distribution with parameter $\alpha$ and $\beta$.

Based on these assumptions the optimal level of base stock system is derived and the variations in the optimal level of base stock with reference to various parameters like holding cost, shortage cost, parameter of inter arrival time distribution and the parameter of demand distribution is studied with the help of numerical illustrations.

$$\frac{d}{h+d} = \sum_{n=1}^{\infty} F_n(B)\left\{\int_0^{\infty}[G_n(Y) - G_{n+1}(Y)]K(Y)dy\right\} \quad (4)$$

$$K(y) = \begin{cases} \theta_1 e^{-\theta_1 y} & ,Y \leq Y_0 \\ \theta_2(y - Y_0)e^{\theta_2(y-Y_0)}e^{-\theta_2 Y_0} & ,Y > Y_0 \end{cases}$$

here, $Y_0 \sim U(\alpha,\beta)$

$$\therefore K(y) = K(y,\theta_1)\ P(Y \leq Y_0) + K(y,\theta_2)\ P(Y \geq Y_0)$$

$$f(y_0) = \frac{1}{\beta-\alpha}\,, \qquad \alpha \le x \le \beta$$

$$P(Y \le Y_0) = P(Y_0 \ge Y) = \int_y^\beta \frac{1}{\beta-\alpha}\,dy_0$$

$$= \frac{1}{\beta-\alpha}[y_0]_y^\beta = \frac{1}{\beta-\alpha}[\beta-y] = \frac{\beta-y}{\beta-\alpha}$$

$$P(Y \ge Y_0) = \int_\alpha^y f(y_0)\,dy_0$$

$$= \theta_1 e^{-\theta_1 y}\frac{\beta-y}{\beta-\alpha} + \theta_2{}^2\int_\alpha^y (Y-Y_0)e^{\theta_2(Y-Y_0)}e^{-\theta_2 Y_0}\frac{1}{\beta-\alpha}\,dy_0 \qquad \mathbf{(5)}$$

Hence ,$\frac{d}{h+d} = \sum_{n=1}^{\infty} F_n(B)\int_0^\infty [G_n(y) - G_{n+1}(y)]K(y)dy$

$$\Rightarrow \sum_{n=1}^{\infty} F_n(B)\left\{\int_0^\infty [G_n(Y) - G_{n+1}(Y)]\left\{\theta_1 e^{-\theta_1 y}\frac{\beta-y}{\beta-\alpha} + \theta_2{}^2\int_\alpha^y (Y-Y_0)e^{\theta_2(Y-Y_0)}e^{-\theta_2 Y_0}\frac{1}{\beta-\alpha}\,dy_0\right\}dy\right\}$$

$$= \sum_{n=1}^{\infty} F_n(B)\left\{\int_0^\infty \frac{[\lambda y]^n}{n!}e^{-\lambda y}\left\{\theta_1 e^{-\theta_1 y}\frac{\beta-y}{\beta-\alpha} + \theta_2{}^2\int_\alpha^y (Y-Y_0)e^{\theta_2(Y-Y_0)}e^{-\theta_2 Y_0}\frac{1}{\beta-\alpha}\,dy_0\right\}dy\right\}$$

$$= \sum_{n=1}^{\infty} F_n(B)\left\{\int_0^\infty \frac{[\lambda y]^n}{n!}e^{-\lambda y}\theta_1 e^{-\theta_1 y}\frac{\beta-y}{\beta-\alpha}\,dy\right.$$
$$\left. + \theta_2{}^2\int_0^\infty \frac{[\lambda y]^n}{n!}e^{-\lambda y}\left[\int_\alpha^y (Y-Y_0)e^{\theta_2(Y-Y_0)}e^{-\theta_2 Y_0}\frac{1}{\beta-\alpha}\,dy_0\right]dy\right\}$$

$$= \sum_{n=1}^{\infty} F_n(B)[I_1 + I_2]$$

consider

$$I_1 = \int_0^\infty \frac{(\lambda y)^n e^{-\lambda y}}{n!}\theta_1 e^{-\theta_1 y}\frac{\beta-y}{\beta-\alpha}\,dy$$
$$= \frac{\theta_1\lambda^n}{\beta-\alpha}\int_0^\infty \frac{y^n}{n!}e^{-(\lambda+\theta_1)y}(\beta-y)dy$$
$$= \frac{\theta_1\lambda^n}{\beta-\alpha}\left\{\beta\int_0^\infty \frac{y^n}{n!}e^{-(\lambda+\theta_1)y}dy - \int_0^\infty \frac{y^{n+1}}{n!}e^{-(\lambda+\theta_1)y}dy\right\}$$
$$= \frac{\theta_1\lambda^n}{\beta-\alpha}\left\{\beta\frac{\Gamma(n+1)}{n!\,(\lambda+\theta_1)^{n+1}} - \frac{\Gamma(n+2)}{n!\,(\lambda+\theta_1)^{n+2}}\right\}$$
$$\therefore I_1 = \frac{\theta_1\lambda^n}{\beta-\alpha}\left\{\frac{\beta}{(\lambda+\theta_1)^{n+1}} - \frac{(n+1)}{(\lambda+\theta_1)^{n+2}}\right\}$$

Consider

$$I_2 = \theta_2\int_0^\infty \frac{(\lambda y)^n e^{-\lambda y}}{n!}\left(\int_\alpha^y (y-y_0)e^{\theta_2(y-y_0)}e^{-\theta_2 y_0}\frac{1}{\beta-\alpha}\,dy_0\right)dy$$

Consider

$$\int_{\alpha}^{y}\left(y-y_0\right)e^{\theta_2\left(y-y_0\right)}e^{-\theta_2 Y_0}\frac{1}{\beta-\alpha}dy_0$$

$$=\frac{1}{\beta-\alpha}\left\{\int_{\alpha}^{y}e^{-2\theta_2 y_0}e^{\theta_2 y}(y-y_0)dy_0\right\}$$

$$=\frac{e^{-\theta_2 y}}{\beta-\alpha}\left\{\int_{\alpha}^{y}ye^{-2\theta_2 y_0}dy_0-\int_{\alpha}^{y}y_0e^{-2\theta_2 y_0}dy_0\right\}$$

$$=\frac{e^{-\theta_2 y}}{\beta-\alpha}\left\{y\left[\frac{e^{-2\theta_2 y_0}}{-2\theta_2}\right]_{\alpha}^{y}-\left[\frac{y_0e^{-2\theta_2 y_0}}{-2\theta_2}\right]_{\alpha}^{y}-\int_{\alpha}^{y}\frac{e^{-2\theta_2 y_0}}{-2\theta_2 y_0}dy_0\right\}$$

$$=\frac{e^{-\theta_2 y}}{\beta-\alpha}\left\{\frac{y}{2\theta_2}\left[e^{-2\theta_2\alpha}-e^{-2\theta_2 y}\right]+\left[\frac{ye^{-2\theta_2 y}-\alpha e^{-2\theta_2\alpha}}{2\theta_2}\right]-\int_{\alpha}^{y}\frac{e^{-2\theta_2 y_0}}{2\theta_2 y_0}dy_0\right\}$$

$$=\frac{e^{-\theta_2 y}}{\beta-\alpha}\left\{\frac{y}{2\theta_2}\left[e^{-2\theta_2\alpha}-e^{-2\theta_2 y}+e^{-2\theta_2 y}-\frac{\alpha}{2\theta_2}e^{-2\theta_2\alpha}-\frac{1}{2\theta_2}\left|\frac{e^{-2\theta_2 y_0}}{-2\theta_2}\right|_{\alpha}^{y}\right]\right\}$$

$$=\frac{e^{-\theta_2 y}}{\beta-\alpha}\left\{\frac{1}{2\theta_2}\left[ye^{-2\theta_2\alpha}-\alpha e^{-2\theta_2\alpha}\right]+\frac{1}{4{\theta_2}^2}\left[e^{-2\theta_2 y}-e^{-2\theta_2\alpha}\right]\right\}$$

$$=\frac{e^{-\theta_2 y}}{\beta-\alpha}\left\{\frac{e^{-2\theta_2\alpha}}{2\theta_2}[y-\alpha]+\frac{1}{4{\theta_2}^2}\left[e^{-2\theta_2 y}-e^{-2\theta_2\alpha}\right]\right\}$$

$$=\frac{e^{-\theta_2 y}}{2\theta_2(\beta-\alpha)}\left\{e^{-2\theta_2\alpha}\left[y-\alpha-\frac{1}{2\theta_2}\right]+\frac{e^{-2\theta_2 y}}{2\theta_2}\right\}$$

Therefore,

$$I_2={\theta_2}^2\int_0^{\infty}\frac{(\lambda y)^n e^{-\lambda y}}{n!}\left\{\frac{e^{-2\theta_2 y}}{2\theta_2(\beta-\alpha)}\left[e^{-2\theta_2\alpha}\left(y-\alpha-\frac{1}{2\theta_2}\right)+\frac{e^{-2\theta_2 y}}{2\theta_2}\right]\right\}dy$$

$$=\frac{\theta_2\lambda^n}{2(\beta-\alpha)}\int_0^{\infty}\frac{y^n}{n!}e^{-y(\lambda+\theta_2)}\left[e^{-2\theta_2\alpha}\left(y-\alpha-\frac{1}{2\theta_2}\right)+\frac{e^{-2\theta_2 y}}{2\theta_2}\right]dy$$

$$=\frac{\theta_2\lambda^n}{2(\beta-\alpha)}\left\{e^{-2\theta_2\alpha}\int_0^{\infty}\frac{y^n}{n!}e^{-y(\lambda+\theta_2)}\left(y-\alpha-\frac{1}{2\theta_2}\right)dy+\frac{1}{2\theta_2}\int_0^{\infty}\frac{y^n}{n!}e^{-y(\lambda+3\theta_2)}dy\right\}$$

$$=\frac{\theta_2\lambda^n}{2(\beta-\alpha)}\left\{\frac{e^{-2\theta_2\alpha}}{n!}\left[\frac{\Gamma(n+2)}{(\lambda+\theta_2)^{n+2}}-\frac{\alpha\,\Gamma(n+1)}{(\lambda+\theta_2)^{n+1}}-\frac{\Gamma(n+1)}{(\lambda+\theta_2)^{n+1}2\theta_2}\right]+\frac{1}{2\theta_2}\frac{\Gamma(n+1)}{n!\,(\lambda+3\theta_2)^{n+1}}\right\}$$

$$=\frac{\theta_2\lambda^n}{2(\beta-\alpha)}\left\{\frac{e^{-2\theta_2\alpha}}{n!}\left[\frac{(n+1)n!}{(\lambda+\theta_2)^{n+2}}-\frac{\alpha n!}{(\lambda+\theta_2)^{n+1}}-\frac{n!}{2\theta_2(\lambda+\theta_2)^{n+1}}\right]+\frac{1}{2\theta_2}\frac{n!}{n!\,(\lambda+3\theta_2)^{n+1}}\right\}$$

$$\therefore I_2=\frac{\theta_2\lambda^n}{2(\beta-\alpha)}\left\{\frac{e^{-2\theta_2\alpha}}{(\lambda+\theta_2)^{n+1}}\left[\frac{(n+1)}{(\lambda+\theta_2)}-\alpha-\frac{1}{2\theta_2}\right]+\frac{1}{2\theta_2(\lambda+3\theta_2)^{n+1}}\right\}$$

Hence,

$$\therefore\frac{d}{h+d}=\sum_{n=1}^{\infty}F_n(B)\left\{\left[\frac{\theta_1\lambda^n}{(\beta-\alpha)}\left\{\frac{\beta}{(\lambda+\theta_1)^{n+1}}-\frac{n+1}{(\lambda+\theta_1)^{n+2}}\right\}\right]\right.$$
$$\left.+\left[\frac{\theta_2\lambda^n}{2(\beta-\alpha)}\left\{\frac{e^{-2\theta_2\alpha}}{(\lambda+\theta_2)^{n+1}}\left[\frac{n+1}{(\lambda+\theta_2)}-\alpha-\frac{1}{2\theta_2}\right]+\frac{1}{2\theta_2(\lambda+3\theta_2)^{n+1}}\right\}\right]\right\}$$

Since the probability distribution of demand at the $i^{th}$ demand epoch is exponential with parameter μ,

$$F_n(B)=\int_0^{B}\frac{\mu^n y^{n-1}e^{-\mu y}}{\Gamma n}dy$$

$$\therefore \frac{d}{h+d} = \sum_{n=1}^{\infty}\int_0^B \frac{\mu^n y^{n-1}e^{-\mu y}}{\Gamma n}\left\{\left[\frac{\theta_1\lambda^n}{(\beta-\alpha)}\left\{\frac{\beta}{(\lambda+\theta_1)^{n+1}}-\frac{n+1}{(\lambda+\theta_1)^{n+2}}\right\}\right]\right.$$

$$\left.+\left[\frac{\theta_2\lambda^n}{2(\beta-\alpha)}\left\{\frac{e^{-2\theta_2\alpha}}{(\lambda+\theta_2)^{n+1}}\left[\frac{n+1}{(\lambda+\theta_2)}-\alpha-\frac{1}{2\theta_2}\right]+\frac{1}{2\theta_2(\lambda+3\theta_2)^{n+1}}\right\}\right]\right\}dy$$

$$= \sum_{n=1}^{\infty}\int_0^B \frac{(\lambda\mu)^n\theta_1 y^{n-1}e^{-\mu y}}{(\beta-\alpha)\,\Gamma n}\left\{\frac{\beta}{(\lambda+\theta_1)^{n+1}}-\frac{n+1}{(\lambda+\theta_1)^{n+2}}\right\}dy$$

$$+\sum_{n=1}^{\infty}\int_0^B \frac{\theta_2(\lambda\mu)^n y^{n-1}e^{-\mu y}}{2(\beta-\alpha)\,\Gamma n}\left\{\frac{e^{-2\theta_2\alpha}}{(\lambda+\theta_2)^{n+1}}\left[\frac{n+1}{(\lambda+\theta_2)}-\alpha-\frac{1}{2\theta_2}\right]+\frac{1}{2\theta_2(\lambda+3\theta_2)^{n+1}}\right\}dy$$

$$= \mathrm{T}_1 + \mathrm{T}_2 \qquad \mathbf{(6)}$$

$$\mathit{Consider}\ \mathrm{T}_1 = \sum_{n=1}^{\infty}\int_0^B \frac{(\lambda\mu)^n\theta_1 y^{n-1}e^{-\mu y}}{(\beta-\alpha)\,\Gamma n}\left\{\frac{\beta}{(\lambda+\theta_1)^{n+1}}-\frac{n+1}{(\lambda+\theta_1)^{n+2}}\right\}dy$$

$$= \frac{\theta_1\beta}{(\beta-\alpha)(\lambda+\theta_1)^{n+1}}\int_0^B\sum_{n=1}^{\infty}\frac{(\lambda\mu)^n y^{n-1}e^{-\mu y}}{\Gamma n}dy - \frac{\theta_1(n+1)}{(\beta-\alpha)(\lambda+\theta_1)^{n+2}}\int_0^B\sum_{n=1}^{\infty}\frac{(\lambda\mu)^n y^{n-1}e^{-\mu y}}{\Gamma n}dy$$

$$= \frac{\theta_1\beta(\lambda\mu)}{(\beta-\alpha)(\lambda+\theta_1)^{n+1}}\int_0^B\sum_{n=1}^{\infty}\frac{(\lambda\mu y)^{n-1}e^{-\mu y}}{(n-1)!}dy - \frac{\theta_1(n+1)(\lambda\mu)}{(\beta-\alpha)(\lambda+\theta_1)^{n+2}}\int_0^B\sum_{n=1}^{\infty}\frac{(\lambda\mu y)^{n-1}e^{-\mu y}}{(n-1)!}dy$$

$$= \frac{\theta_1\beta(\lambda\mu)}{(\lambda-\alpha)(\lambda+\theta_1)^{n+1}}\left[\int_0^B e^{\lambda\mu y-\mu y}dy\right] - \frac{\theta_1(n+1)(\lambda\mu)}{(\beta-\alpha)(\lambda+\theta_1)^{n+2}}\left[\int_0^B e^{\lambda\mu y-\mu y}dy\right]$$

$$= \frac{\theta_1\beta(\lambda\mu)}{(\beta-\alpha)(\lambda+\theta_1)^{n+1}}\left[\int_0^B e^{-y(\mu-\lambda\mu)}dy\right] - \frac{\theta_1(n+1)(\lambda\mu)}{(\beta-\alpha)(\lambda+\theta_1)^{n+2}}\left[\int_0^B e^{-y(\mu-\lambda\mu)}dy\right]$$

$$= \frac{\theta_1\beta(\lambda\mu)}{(\beta-\alpha)(\lambda+\theta_1)^{n+1}}\left[\left|\frac{e^{-y(\mu-\lambda\mu)}}{-(\mu-\lambda\mu)}\right|_0^B\right] - \frac{\theta_1(n+1)(\lambda\mu)}{(\beta-\alpha)(\lambda+\theta_1)^{n+2}}\left[\left|\frac{e^{-y(\mu-\lambda\mu)}}{-(\mu-\lambda\mu)}\right|_0^B\right]$$

$$= \frac{\theta_1\beta(\lambda\mu)}{(\beta-\alpha)(\lambda+\theta_1)^{n+1}}\left[\frac{1-e^{-B\mu(1-\lambda)}}{\mu(1-\lambda)}\right] - \frac{\theta_1(n+1)(\lambda\mu)}{(\beta-\alpha)(\lambda+\theta_1)^{n+2}}\left[\frac{1-e^{-B\mu(1-\lambda)}}{\mu(1-\lambda)}\right]$$

$$= \frac{\theta_1\beta\lambda\left(1-e^{-B\mu(1-\lambda)}\right)}{(\beta-\alpha)(\lambda+\theta_1)^{n+1}(1-\lambda)} - \frac{\theta_1(n+1)\lambda}{(\beta-\alpha)(\lambda+\theta_1)^{n+2}}\left[\frac{1-e^{-B\mu(1-\lambda)}}{(1-\lambda)}\right]$$

$$\therefore T_1 = \frac{\lambda\theta_1\left(1-e^{-B\mu(1-\lambda)}\right)}{(1-\lambda)(\beta-\alpha)}\left[\frac{\beta}{(\lambda+\theta_1)^{n+1}}-\frac{(n+1)}{(\lambda+\theta_1)^{n+2}}\right] \qquad \mathbf{(7)}$$

$$\mathit{Consider}\ T_2 = \sum_{n=1}^{\infty}\int_0^B \frac{\theta_2(\lambda\mu)^n y^{n-1}e^{-\mu y}}{2(\beta-\alpha)\,\Gamma n}\left\{\frac{e^{-2\theta_2\alpha}}{(\lambda+\theta_2)^{n+1}}\left[\frac{(n+1)}{(\lambda+\theta_2)}-\alpha-\frac{1}{2\theta_2}\right]+\frac{1}{2\theta_2(\lambda+3\theta_2)^{n+1}}\right\}dy$$

$$= \sum_{n=1}^{\infty}\int_0^B \frac{\theta_2(\lambda\mu)^n y^{n-1}e^{-\mu y}}{2(\beta-\alpha)\,\Gamma n}\frac{e^{-2\theta_2\alpha}}{(\lambda+\theta_2)^{n+1}}\left[\frac{(n+1)}{(\lambda+\theta_2)}-\alpha-\frac{1}{2\theta_2}\right]dy$$

$$+\ \sum_{n=1}^{\infty}\int_0^B \frac{\theta_2(\lambda\mu)^n y^{n-1}e^{-\mu y}}{2(\beta-\alpha)\,\Gamma n}\frac{1}{2\theta_2(\lambda+3\theta_2)^{n+1}}dy$$

$$= \frac{\theta_2 \lambda \mu e^{-2\theta_2 \lambda}}{2(\beta-\alpha)(\lambda+\theta_2)^{n+1}} \left[\frac{(n+1)}{(\lambda+\theta_2)} - \alpha - \frac{1}{2\theta_2}\right] \int_0^B \sum_{n=1}^{\infty} \frac{(\lambda \mu y)^{n-1} e^{-\mu y}}{(n-1)!} dy$$

$$+ \frac{(\lambda\mu)}{2(\beta-\alpha)(\lambda+3\theta_2)^{n+1}} \int_0^B \sum_{n=1}^{\infty} \frac{(\lambda \mu y)^{n-1} e^{-\mu y}}{(n-1)!} dy$$

$$= \frac{\theta_2 \lambda \mu e^{-2\theta_2 \alpha}}{2(\beta-\alpha)(\lambda+\theta_2)^{n+1}} \left[\frac{(n+1)}{(\lambda+\theta_2)} - \alpha - \frac{1}{2\theta_2}\right] \int_0^B e^{\lambda \mu y - \mu y} dy + \frac{\lambda\mu}{2(\beta-\alpha)(\lambda+3\theta_2)^{n+1}} \int_0^B e^{\lambda \mu y - \mu y} dy$$

$$= \frac{\theta_2 \lambda \mu e^{-2\theta_2 \alpha}}{2(\beta-\alpha)(\lambda+\theta_2)^{n+1}} \left[\frac{(n+1)}{(\lambda+\theta_2)} - \alpha - \frac{1}{2\theta_2}\right] \left[\left|\frac{e^{-\mu(1-\lambda)y}}{-\mu(1-\lambda)}\right|_0^B\right] + \frac{\lambda\mu}{2(\beta-\alpha)(\lambda+3\theta_2)^{n+1}} \left[\frac{e^{-\mu(1-\lambda)y}}{-\mu(1-\lambda)}\right]$$

$$= \frac{\theta_2 \lambda e^{-2\theta_2 \alpha}}{2(\beta-\alpha)(\lambda+\theta_2)^{n+1}} \left[\frac{(n+1)}{(\lambda+\theta_2)} - \alpha - \frac{1}{2\theta_2}\right] \left[\frac{1-e^{-\mu(1-\lambda)B}}{(1-\lambda)}\right] + \frac{\lambda}{2(\beta-\alpha)(\lambda+3\theta_2)^{n+1}} \left[\frac{1-e^{-\mu(1-\lambda)B}}{(1-\lambda)}\right]$$

$$\therefore T_2 = \left[\frac{\theta_2 \lambda e^{-2\theta_2 \alpha}}{2(\beta-\alpha)(\lambda+\theta_2)^{n+1}} \left(\frac{(n+1)}{(\lambda+\theta_2)} - \alpha - \frac{1}{2\theta_2}\right) + \frac{\lambda}{2(\beta-\alpha)(\lambda+3\theta_2)^{n+1}}\right] \left[\frac{1-e^{-\mu(1-\lambda)B}}{(1-\lambda)}\right] \quad \textbf{(8)}$$

$$\therefore \frac{d}{h+d} = \left[\frac{\lambda\theta_1\left(1-e^{-\mu(1-\lambda)B}\right)}{(1-\lambda)(\beta-\alpha)}\right] \left\{\left[\frac{\beta}{(\lambda+\theta_1)^{n+1}} - \frac{(n+1)}{(\lambda+\theta_1)^{n+2}}\right] + \left[\frac{\theta_2 \lambda e^{-2\theta_2 \alpha}}{2(\beta-\alpha)(\lambda+\theta_2)^{n+1}} \left(\frac{(n+1)}{(\lambda+\theta_2)} - \alpha - \frac{1}{2\theta_2}\right) + \frac{\lambda}{2(\beta-\alpha)(\lambda+3\theta_2)^{n+1}}\right] \left[\frac{1-e^{-\mu(1-\lambda)B}}{(1-\lambda)}\right]\right\}$$

$$\therefore \frac{d}{h+d} = \left[\frac{\lambda\left(1-e^{-\mu(1-\lambda)B}\right)}{(1-\lambda)(\beta-\alpha)}\right] \left\{\left[\frac{\theta_1\beta}{(\lambda+\theta_1)^{n+1}} - \frac{\theta_1(n+1)}{(\lambda+\theta_1)^{n+2}}\right] + \left[\frac{\theta_2 e^{-2\theta_2 \alpha}}{2(\lambda+\theta_2)^{n+1}} \left(\frac{(n+1)}{(\lambda+\theta_2)} - \alpha - \frac{1}{2\theta_2}\right) + \frac{\lambda}{2(\lambda+3\theta_2)^{n+1}}\right]\right\} \quad \textbf{(9)}$$

For the fixed values of h, d, $\lambda$, $\beta$, $\mu$, $\theta_1$, $\theta_2$, $\alpha$ and n, the optimal value of B, can be obtained.

## 6. NUMERICAL ILLUSTRATIONS

The variations in the values of optimal Base stock" $\hat{B}$ " ,consequent on the changes in the parametersh, d, $\lambda$ and μ have been studied by taking numerical illustrations. The tables and the corresponding graphs are given.

### *6.1. Case(i)*

For the fixed values of d=3, $\lambda$=2, β=6, $\alpha$ =4, $\theta_1$ = 4, $\theta_2$=1, μ=1.5, and n = 5, the optimal value of B is obtained when the variations of h.

Table1.The variations in $\hat{B}$ for the changes in the value of h.

| **h** | **5** | **10** | **15** | **20** |
|---|---|---|---|---|
| **B** | 3.5811 | 3.2590 | 3.044 | 2.883 |

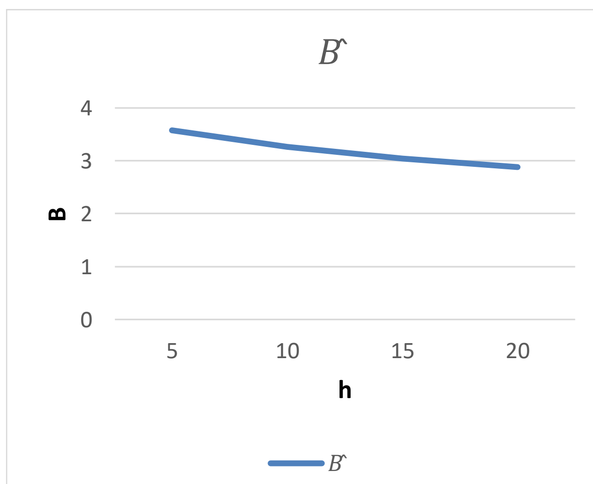


Fig. 1.The variations in $\hat{B}$for the changes in the value of h

### *6.2. Case(ii)*

For the fixed values of h = 5, $\lambda$=2, β=6, $\alpha$ =4, $\theta_1$ = 4, $\theta_2$=1, μ=1.5, and n = 5, the optimal value of B is obtained when the variations of d.

Table 2. The variations in $\hat{B}$for the changes in the value ofd.

| **d** | **3** | **6** | **9** | **12** |
|---|---|---|---|---|
| **B** | 3.5811 | 3.83 | 3.94 | 4.002 |

### *6.3. Case(iii)*

For the fixed values of h = 5, d= 3, β=6, $\alpha$ =4, $\theta_1$ = 4, $\theta_2$=1, μ=1.5, and n = 5, the optimal value of B is obtained when the variations of $\lambda$.

Table 3. The variations in $\hat{B}$ for the changes in the value of λ.

| $\boldsymbol{\lambda}$ | **2** | **2.5** | **3** | **3.5** |
|---|---|---|---|---|
| **B** | 3.5811 | 2.742 | 2.263 | 1.94 |

### *6.4. Case(iv)*

For the fixed values of h = 5, d= 3, $\lambda = 2$, β=6, $\alpha$ =4, $\theta_1$ = 4, $\theta_2$=1, and n = 5, the optimal value of B is obtained when the variations of $\mu$.

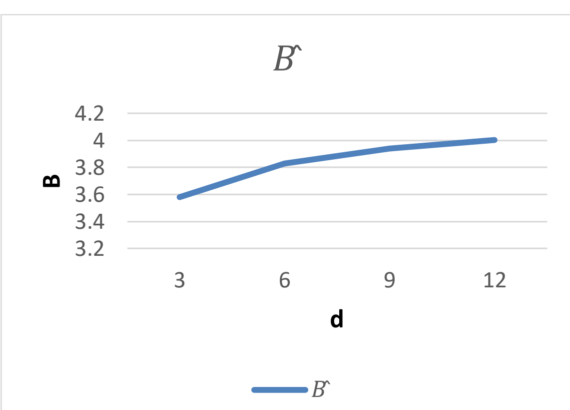


Fig. 2.The variations in $\widehat{B}$ for the changes in the value of d.

Table 4. The variations in $\hat{B}$ for the changes in the value of μ.

| μ | **1.5** | **2** | **2.5** | **3** |
|---|---|---|---|---|
| **B** | 3.5811 | 2.686 | 2.149 | 1.791 |

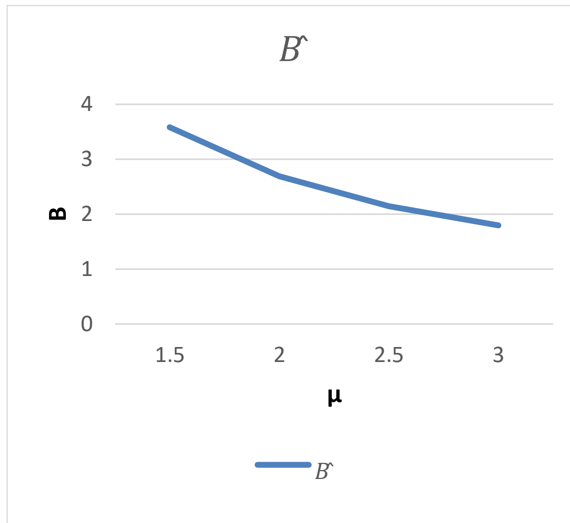


Fig.4.The variations in $\hat{B}$ for the changes in the value of $\mu$.

## 7. FINDINGS AND CONCLUSIONS

From the tables and graphs, it is observed that,

As the holding cost (h) increases, the optimal Base Stock $\hat{B}$ decreases. Hence it suggests less base stock.

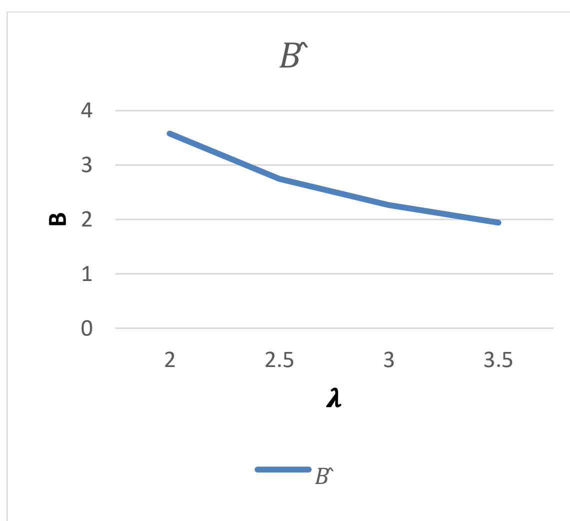


Fig. 3. The variations in $\hat{B}$ for the changes in the value of λ.

As the shortage cost (d) increases, the optimal Base Stock $\hat{B}$ increases and it tells us to maintain the more base stock level.

As $(\lambda)$, the parameter of inter arrival time distribution between demand epochs increases, the optimal Base Stock $\hat{B}$ decreases. It is inferred that the total demand is distributed at different demand epochs with lesser demand and it suggest lesser base stock level.

As $(\mu)$, the parameter of demand distribution increases, the optimal Base Stock $\hat{B}$ decreases. It is inferred that the average demand is distributed at different demand epochs with lesser demand and it suggest lesser base stock level.

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